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by

B.K. Sinha and H.F. Tiersten

Office of Naval Research
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Technical Report No. 30

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The first temperature derivatives of the fundamental elastic constants of quartz are employed along with the thermally-induced biasing strains in the equation for the first perturbation of the eigenvalue for the linear electroelastic equations for small fields superposed on a bias to calculate the resulting change in surface wave velocity with temperature. Since the description employed is referred to a fixed reference state, the geometry does not change and the temperature coefficient of velocity is the negative

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ON THE TEMPERATURE DEPENDENCE OF THE VELOCITY OF SURFACE WAVES IN QUARTZ

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ABSTRACT

The first temperature derivatives of the fundamental elastic constants of quartz are employed along with the thermally-induced biasing strains in the equation for the first perturbation of the eigenvalue for the linear electroelastic equations for small fields superposed on a bias to calculate the resulting change in surface wave velocity with temperature. Since the description employed is referred to a fixed reference state, the geometry does not change and the temperature coefficient of velocity is the negative of the temperature coefficient of delay. In all earlier work a variable temperature dependent geometric state was considered, but the attendant shearing or skewing of the coordinate axes was not properly included. The temperature dependence of the velocity of surface waves propagating in various directions on various cuts of quartz is obtained. For cases in which measurements are available the calculations are shown to be in substantially better agreement with the measurements than previous calculations.

* Work performed while at Rensselaer Polytechnic Institute

1. Introduction

Previous work[1,2] on the temperature dependence of the velocity of surface waves in quartz has employed the temperature derivatives of the effective elastic constants of quartz, which are referred to the variable temperature dependent intermediate position rather than the fixed reference position, to which the fundamental elastic constants are referred. Not only does the intermediate coordinate system extend or contract under a temperature change, but it shears or skews as well. This shearing or skewing of the axes was omitted in the original determination[3] of the temperature derivatives of the effective elastic constants. Although, in the existing treatments[1,2] of the temperature dependence of surface wave velocity, the change in density and path length were properly included, the shearing or skewing of the coordinate axes was omitted.

Recently, the first temperature derivatives of the fundamental elastic constants of quartz were determined[4] from the original data[3] from which the temperature derivatives of the effective constants had been obtained[3]. Also, a perturbation analysis of the linear electroelastic equations for small fields superposed on a bias[5] has been performed[6] and the equation for the first perturbation of the eigenvalue has been obtained. The aforementioned first temperature derivatives of the fundamental elastic constants of quartz are employed along with the thermally-induced biasing strains in the equation for the first perturbation of the eigenvalue[6] to calculate the resulting change in surface wave velocity with temperature.

Since the description employed is referred to the <u>fixed</u> reference state at the reference temperature, the mass density and geometry do not change and the temperature coefficient of natural velocity, which we determine, is the negative of the temperature coefficient of delay. The temperature dependence of the actual, in addition to that of the natural, velocity of surface waves on a number of cuts of quartz is obtained and compared with the previous calculations as well as measured values. In addition, the temperature coefficient of delay and the associated power flow angle are calculated as a function of propagation direction for two interesting orientations of quartz substrates. All results obtained here are limited to linear behavior with temperature because only the first temperature derivatives of the fundamental elastic constants of quartz are known[4]. As noted in an earlier work[7] the surface wave velocity can also be obtained simply by solving the appropriate linear surface wave boundary value problem in the usual manner at both the reference temperature and present temperature when the temperature dependent constants have been determined at the present temperature. The perturbation procedure is being employed here because it enables the <u>change</u> in surface wave velocity to be calculated <u>directly</u>, thereby resulting in greater accuracy for the same number of significant figures and a substantial reduction in computer time.

2. <u>Linear Electroelastic Equations for Small Fields Superposed on a Thermally Induced Bias</u>

The linear electroelastic equations for small fields superposed on a bias may be written in the form[5,6]

$$\tilde{K}_{L\gamma,L} = \rho^{\circ}\ddot{u}_{\gamma}, \quad \tilde{\mathcal{D}}_{L,L} = 0, \tag{2.1}$$

where, since only mechanical nonlinearities are of interest here,

$$\begin{aligned} \tilde{K}_{L\gamma} &= \mathcal{G}_{L\gamma M\nu} u_{\nu,M} + e_{ML\gamma}\tilde{\varphi}_{,M}, \\ \tilde{\mathcal{D}}_L &= e_{LM\nu} u_{\nu,M} - \epsilon_{LM}\tilde{\varphi}_{,M}, \end{aligned} \tag{2.2}$$

and

$$\underset{1}{G}_{L\gamma M\nu} = \underset{2}{c}_{L\gamma M\nu} + \hat{c}_{L\gamma M\nu} + (d\underset{2}{c}_{L\gamma M\nu}/dT)(T - T_o) ,$$

$$\hat{c}_{L\gamma M\nu} = T^1_{LM}\delta_{\gamma\nu} + \underset{3}{c}_{L\gamma M\nu AB}E^1_{AB} + \underset{2}{c}_{L\gamma KM}w_{\nu,K} + \underset{2}{c}_{LKM\nu}w_{\gamma,K} . \tag{2.3}$$

Equations (2.1) constitute the stress equations of motion and charge equation of electrostatics referred to the position coordinates of material points before the thermally induced static deformation is applied, which are called reference coordinates and are denoted by X_M. Equations (2.2) are the linear electroelastic constitutive equations and Eqs. (2.3) contain the definitions associated with the effective elastic coefficients appearing in (2.2). In (2.1) - (2.3) $\tilde{K}_{L\gamma}$, $\tilde{\mathcal{D}}_L$ and u_γ denote the components of the small field Piola-Kirchhoff stress tensor which is asymmetric, reference electric displacement vector, and mechanical displacement vector, respectively; ρ^o and $\tilde{\varphi}$ denote the reference mass density and small field electric potential, respectively, $\underset{2}{c}_{L\gamma M\nu}$ and $\underset{3}{c}_{L\gamma M\nu AB}$ denote the second and third order elastic constants, respectively, $e_{LM\nu}$ and ϵ_{LM} denote the piezoelectric and dielectric constants, respectively, and T_o and T denote the reference and present temperature, respectively.

In this description the present position $\underset{\sim}{y}$ of material points is related to the reference position $\underset{\sim}{X}$ by

$$\underset{\sim}{y}(X_L, t) = \underset{\sim}{X} + \underset{\sim}{w}(X_L) + \underset{\sim}{u}(X_L, t) , \tag{2.4}$$

where $\underset{\sim}{w}$ denotes the displacement due to the thermally induced static deformation. In (2.3) T^1_{LM} and E^1_{AB} denote the components of the thermally induced static biasing stress and strain, respectively, and for small strains and changes in temperature from the reference temperature T_o, we have[4]

$$E^1_{AB} = \frac{1}{2}\,(w_{A,B} + w_{B,A})\,, \tag{2.5}$$

$$T^1_{LM} = \underset{2}{c}_{LMRS} E^1_{RS} - \nu_{LM}(T - T_o)\,, \tag{2.6}$$

where ν_{LM} denotes the thermoelastic coupling coefficients, which are related to the usual coefficients of linear thermal expansion α_{JK} by

$$\nu_{LM} = c_{LMJK}\alpha_{JK}\,. \tag{2.7}$$

The upper cycle notation for many dynamic variables and the capital Latin and lower case Greek index notation are being employed for consistency with the notation of Refs.5 and 6, as is the remainder of the notation in this section. The fact that the capital Latin and lower case Greek indices refer to the reference and intermediate position coordinates, respectively, is not important here and in this work they may be used interchangeably. We employ Cartesian tensor notation, the summation convention for repeated tensor indices, the convention that a comma followed by a Latin index denotes partial differentiation with respect to reference coordinates and the dot notation for differentiation with respect to time. Since we are considering a homogeneous stress free biasing state resulting from a homogeneous temperature change $(T - T_o)$, from (2.6) and (2.7) we obtain

$$E^1_{RS} = \alpha_{RS}(T - T_o)\,. \tag{2.8}$$

Since it has been shown[4] that a static homogeneous infinitesimal rigid rotation has no influence on any results, without any loss in generality we may choose the homogeneous rigid rotation to vanish and substitute from (2.8) into $(2.3)_2$ to obtain

$$\hat{c}_{L\gamma M\nu} = (\underset{3}{c}_{L\gamma M\nu AB}\alpha_{AB} + \underset{2}{c}_{L\gamma KM}\alpha_{\nu K} + \underset{2}{c}_{LKM\nu}\alpha_{\gamma K})(T - T_o)\,. \tag{2.9}$$

3. Perturbation Equations

For purely elastic nonlinearities, which are the only ones of interest here, the equation for the first perturbation of the eigenvalue obtained from the perturbation analysis[8] mentioned in the Introduction may be written in the form

$$\Delta V/V_1 = H_1/2\xi^2 V_1^2, \quad V = V_1 + \Delta V, \tag{3.1}$$

where V_1 and V are the unperturbed abd perturbed surface wave velocities at T_o and T, respectively, ξ is the unperturbed propagation wavenumber and

$$H_1 = + \int_R \tilde{K}^n_{L\gamma} g_{\gamma,L}\, dV, \tag{3.2}$$

where R is the volume enclosed in a wavelength in a unit length of the surface wave from the top of the substrate down. In (3.2) g_γ denotes the normalized mechanical displacement vector, and $\tilde{K}^n_{L\gamma}$ denotes the portion of the Piola-Kirchhoff stress tensor resulting from the biasing state and the change in the elastic constants $\Delta c_{2L\gamma M\alpha}$ with temperature in the presence of the g_γ, and is given by

$$\tilde{K}^n_{L\gamma} = (\hat{c}_{L\gamma M\alpha} + \Delta c_{2L\gamma M\alpha}) g_{\alpha,M}, \tag{3.3}$$

where

$$\Delta c_{2L\gamma M\alpha} = (dc_{2L\gamma M\alpha}/dT)(T - T_o), \tag{3.4}$$

and the $dc_{2L\gamma M\alpha}/dT$ are the first temperature derivatives of the fundamental elastic constants of quartz, which are tabulated in Ref. 4. The normalized eigensolution g_γ and $\tilde{f}$ is defined by

$$g_\gamma = \frac{u_\gamma}{N}, \quad \tilde{f} = \frac{\tilde{\varphi}}{N}, \quad N^2 = \int_V \rho^o u_\gamma u_\gamma\, dV, \tag{3.5}$$

where u_γ and $\tilde{\varphi}$ are the mechanical displacement and electric potential, respectively, which satisfy the equations of linear piezoelectricity, i.e., Eqs. (2.1) and (2.2), with (2.3) and (2.9), at $T = T_0$ subject to the boundary conditions

$$N_L \tilde{K}_{L\gamma} = 0\,,\quad -N_L \tilde{\mathcal{D}}_L \doteq \epsilon_0 \xi \tilde{\varphi}\,,\quad \text{at } N_L X_L = 0\,, \tag{3.6}$$

at $T = T_0$, the first of which, with $(2.2)_1$, (2.3) and (2.9), takes the form

$$N_L (c_{2L\gamma M\nu} u_{\nu,M} + e_{ML\gamma} \tilde{\varphi}_{,M}) = 0\,. \tag{3.7}$$

The second equation in (3.6) is a consequence[9] of Laplace's equation in free space, the two electrical continuity conditions at the surface of the substrate and the form of the surface wave solution appearing in (4.1). In (3.6) and (3.7) N_L denotes the unit normal to the free surface of the substrate at $T = T_0$ and ϵ_0 is the permittivity of free space.

4. Temperature Dependence of Surface Wave Velocity

A schematic diagram of the free surface of the substrate at the reference temperature $T = T_0$ is shown in Fig.1. The solution for surface waves propagating in the X_τ-direction may be written in the form

$$(u_\gamma, \tilde{\varphi}) = \sum_{m=1}^{4} C^{(m)} (A_\gamma^{(m)}, B^{(m)}) e^{i\beta_m \xi X_\nu} e^{i\xi(X_\tau - V_1 t)}\,, \tag{4.1}$$

where the unperturbed surface wave velocity V_1 at $T = T_0$ is determined numerically by trial and error so that (2.1) and (3.6), with (2.2), (2.3) and (2.9), at $T = T_0$ are satisfied[10-12]. When V_1 is obtained the equations determine the β_m, $A_\gamma^{(m)}$, $B^{(m)}$ and $C^{(m)}$. Since the straight-crested surface wave solution depends only on the two coordinates X_τ and X_ν, for integration over a wavelength Eq. (3.2) takes the somewhat reduced form

$$H_1 = -\int_0^\infty dX_\nu \int_{-\pi/\xi}^{\pi/\xi} dX_\tau \Big[\tilde{K}^n_{\tau\tau} g_{\tau,\tau} + \tilde{K}^n_{\tau\nu} g_{\nu,\tau} + \tilde{K}_{\tau\sigma} g_{\sigma,\tau} + \tilde{K}^n_{\nu\tau} g_{\tau,\nu} + \tilde{K}_{\nu\nu} g_{\nu,\nu} + \tilde{K}_{\nu\sigma} g_{\sigma,\nu}\Big], \tag{4.2}$$

where X_σ denotes the axis normal to X_τ and X_ν and in (4.2) we have introduced the convention that repeated Greek indices are not to be summed. For this case of surface waves the normalization integral in $(3.5)_3$ takes the form

$$N^2 = \rho^o \int_0^\infty dX_\nu \int_{-\pi/\xi}^{\pi/\xi} u_\gamma u_\gamma \, dX_\tau = \frac{\rho^o \pi i}{\xi^2} \sum_{m=1}^{4} \sum_{n=1}^{4} \frac{C^{(m)} A_\gamma^{(m)} C^{(n)*} A_\gamma^{(n)*}}{(\beta_m - \beta_n^*)}, \tag{4.3}$$

where the * denotes complex conjugate. The substitution of (4.1) and (4.3) into $(3.5)_1$ which, with (2.9) and (3.4), is then substituted into (3.3), which is then substituted along with $(3.5)_1$ into (4.2), the integration of which yields the expression for H_1, which enables the change in surface wave velocity to be calculated from (3.1). The resulting expression for H_1 is extremely lengthy and not terribly revealing. Consequently, we do not bother to present it here. Nevertheless, it is clear from (4.2) and (3.3), with (3.4) and (2.9), along with $(3.5)_1$, (4.1) and (4.3) that the resulting expression for H_1 is linear in $(T - T_o)$, depends on the second[13] and third[14] order elastic constants, the coefficients of linear expansion[15] and the recently determined[4] first temperature derivatives of the fundamental elastic constants of quartz, as well as on the known surface wave solution at $T = T_o$, which depends on the linear piezoelectric constants[13].

The change in velocity ΔV determined from (3.1) is the change in natural[16] velocity, which is related to the measured unchanged geometry at $T = T_o$. Since it is not feasible to continually measure the changing lengths with temperature and time intervals are usually measured rather than the actual velocity of the wave, the natural velocity[16] is the most purposeful one to define. However, since previous workers have employed the change in actual velocity calculated from the measurements, we will calculate the change in actual velocity from the change in natural velocity. The relation between them is[17]

$$\Delta V_a/V_1 = \Delta V/V_1 + E^1_{\tau\tau}, \quad \tau \text{ no sum}, \tag{4.4}$$

where the repeated Greek indices in (4.4) are not to be summed and ΔV_a is the change in actual velocity as defined in the Appendix. Since reference coordinates and natural velocity are employed here, the geometry remains fixed and the temperature coefficient of delay (TCD) is given by

$$\mathrm{TCD} = \Delta\tau/\tau(T - T_o) = -\Delta V/V_1(T - T_o), \tag{4.5}$$

where τ is the delay time. Equation (4.5) shows that when reference coordinates and natural velocity are employed, the temperature coefficient of delay is simply the negative of the temperature coefficient of velocity. Substituting from (4.4) into (4.5) and employing (2.8), we obtain

$$\mathrm{TCD} = -\Delta V_a/V_1(T - T_o) + \alpha_{\tau\tau}, \quad \tau \text{ no sum}, \tag{4.6}$$

which is the well-known usual relation between the TCD and the temperature coefficient of actual velocity referred to the variable temperature dependent intermediate configuration of the crystal.

Calculations have been performed for various propagation directions on various orientations of quartz substrates and the results are plotted in Figs.2 - 8. Figures 2 and 3 show the TCD and $\Delta V_a/V_1(T - T_o)$ as a function of propagation direction for surface waves on AT-cut and AC-cut quartz substrates, respectively, at 25°C. The dotted curve shows the average of the calculated values at 0°C and 50°C from Ref.1 and the circles are the average of the experimental values at 0°C and 50°C also from Ref.1. It is clear from

Figs.2 and 3 that the calculations performed here using the proper nonlinearly based formalism are in substantially better agreement with the experimental data of Ref.1 than the calculations performed in Ref.1 using an incomplete linearly based description. Figure 4 shows the calculated TCD for surface waves propagating along the digonal axis of rotated Y-cut quartz substrates as a function of rotation angle at 25°C. The dotted curve is obtained from the calculations of Ref.1 in the same manner as in Figs.2 and 3. Figures 5 and 6 show the calculated TCD as a function of propagation direction for surface waves on X-cut and Y-cut quartz substrates, respectively, at 25°C. The dotted curves are obtained from the calculations of Ref.1 in the same manner as in Figs.2 and 3. Figures 7 and 8 show the temperature coefficient of delay and associated power flow angle as a function of propagation direction for two particularly interesting orientations of doubly-rotated quartz substrates, namely those having[18] $\varphi = 30^\circ$, $\theta = 34.25^\circ$ and $\varphi = 5^\circ$, $\theta = 47.7^\circ$, respectively.

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FIGURE CAPTIONS

Figure 1 Schematic Diagram Showing the Free Surface of a Semi-Infinite Solid

Figure 2 Temperature Coefficients of Actual Velocity and Delay for Surface Waves on AT-Cut Quartz as a Function of Propagation Direction Relative to the Digonal Axis at 25°C. The dotted curve shows the average of the calculated values at 0°C and 50°C from Ref.1. The circles are the average of the experimental values at 0°C and 50°C from Ref.1

Figure 3 Temperature Coefficient of Actual Velocity and Delay for Surface Waves on AC-Cut Quartz as a Function of Propagation Direction Relative to the Digonal Axis at 25°C. The notation convention is the same as in Fig.2.

Figure 4 Temperature Coefficient of Delay for Surface Waves Propagating along the Digonal Axis of Rotated Y-Cuts of Quartz as a Function of Rotation Angle at 25°C. The notation convention is the same as in Fig.2.

Figure 5 Temperature Coefficient of Delay for Surface Waves on X-Cut Quartz as a Function of Propagation Direction Relative to the Y-Axis at 25°C. The notation convention is the same as in Fig.2.

Figure 6 Temperature Coefficient of Delay for Surface Waves on Y-Cut Quartz as a Function of Propagation Direction Relative to the X-Axis at 25°C. The notation convention is the same as in Fig.2.

Figure 7 Temperature Coefficient of Delay and Power Flow Angle as a Function of Propagation Direction Relative to the Axis of Second Rotation at 25°C for the Doubly-Rotated Cut of Quartz Having $\varphi = 5^\circ$, $\theta = 47.7^\circ$.

Figure 8 Temperature Coefficient of Delay and Power Flow Angle as a Function of Propagation Direction Relative to the Axis of Second Rotation at 25°C for the Doubly-Rotated Cut of Quartz Having $\varphi = 30^\circ$, $\theta = 34.25^\circ$.

Figure 9 Schematic Diagram Showing the Relation Between the Surface Normal and Surface Wave Propagation Direction and Wavelength in the Undeformed Reference State at Temperature T_o and the Deformed State at Temperature T.

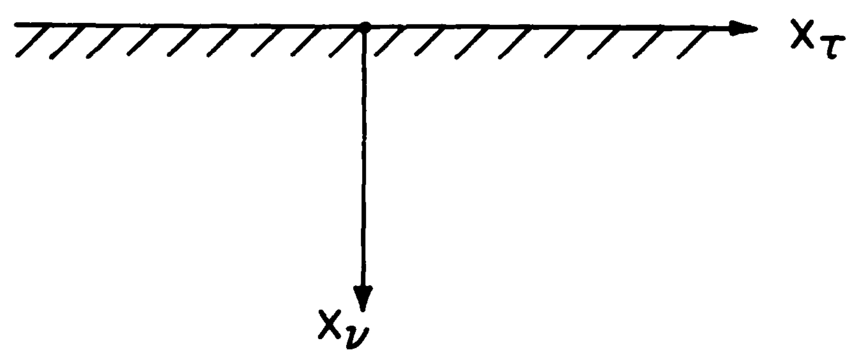
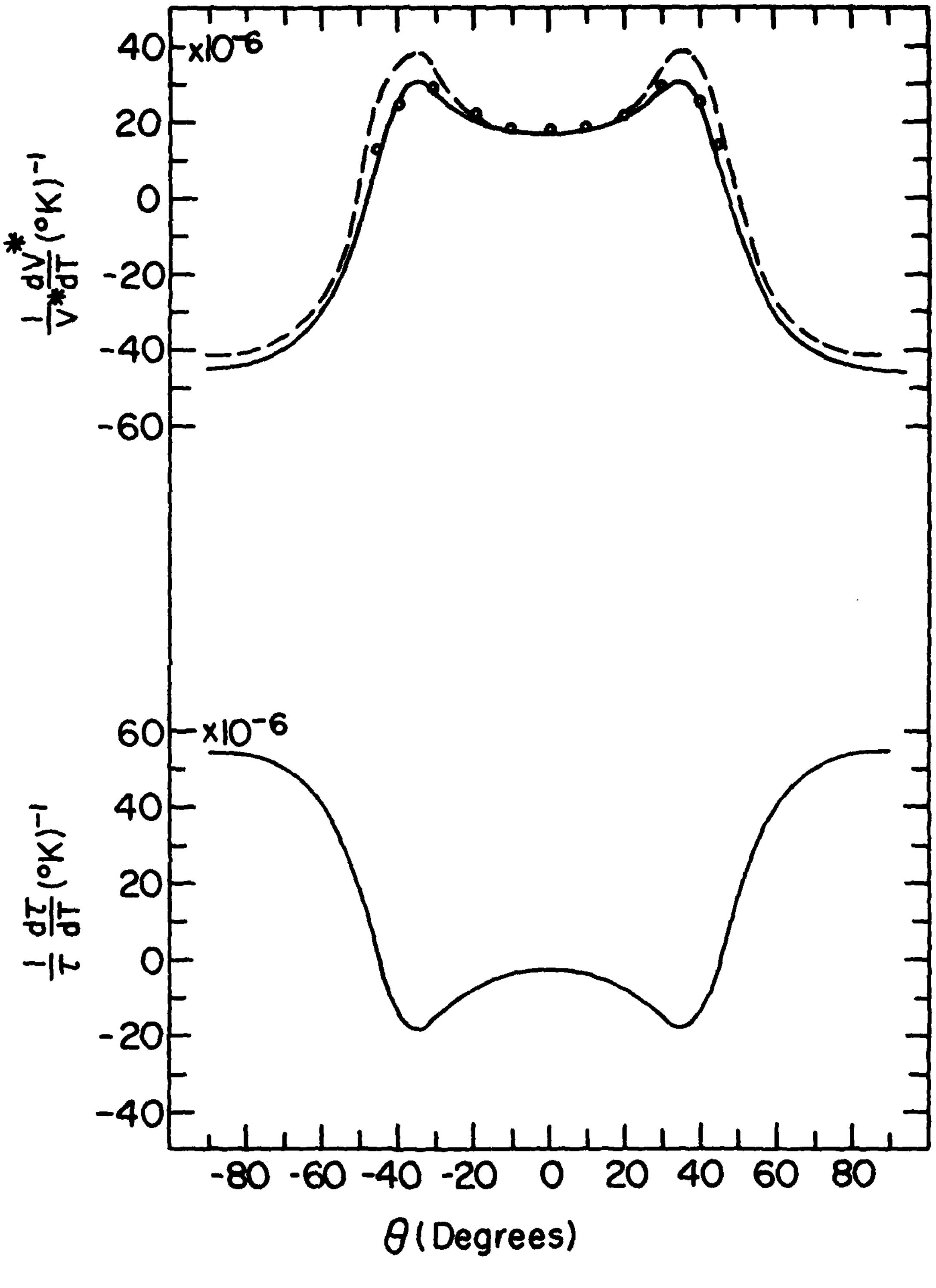


Figure 1



40
20
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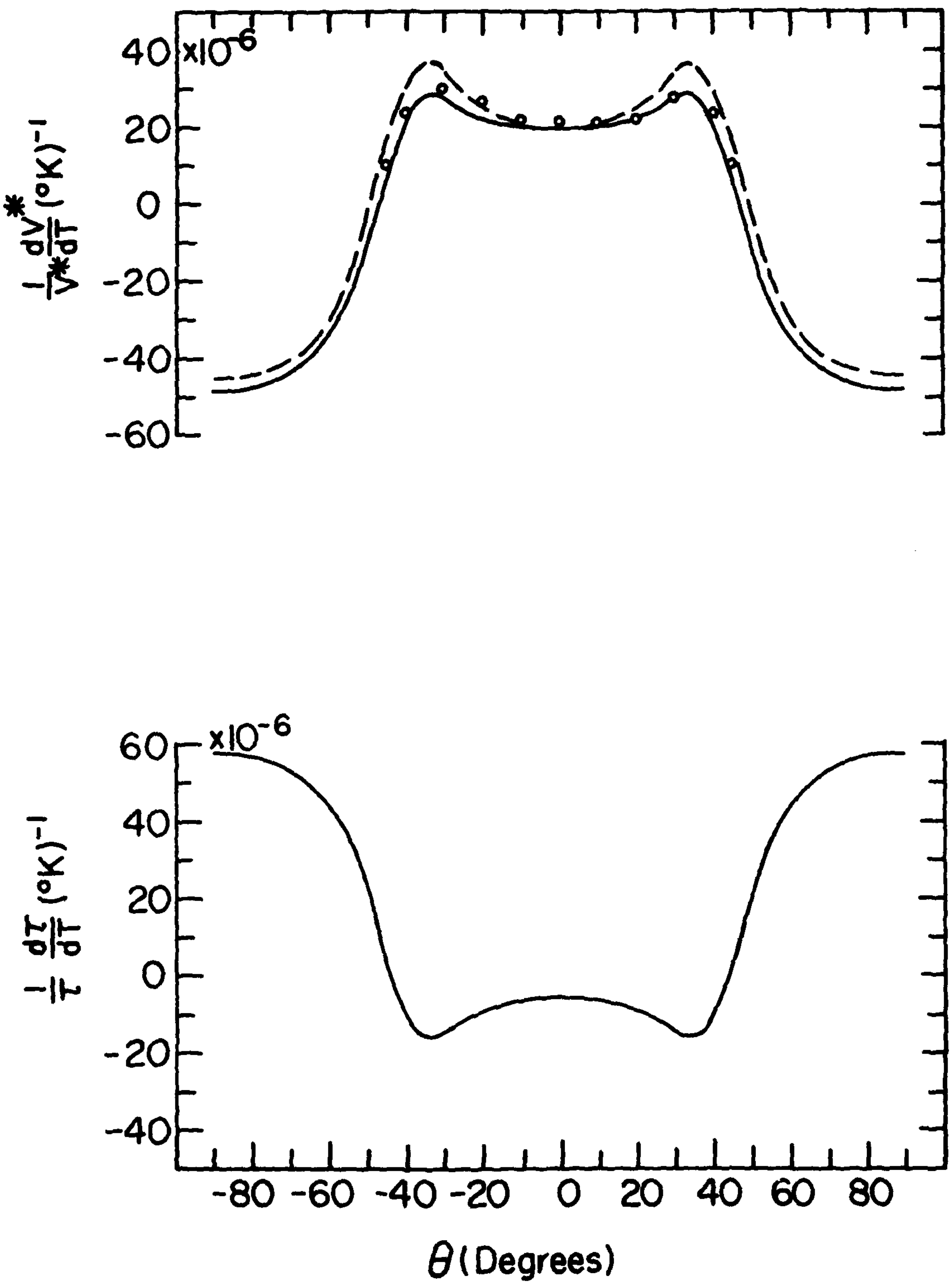
θ (Degrees)

Figure 2



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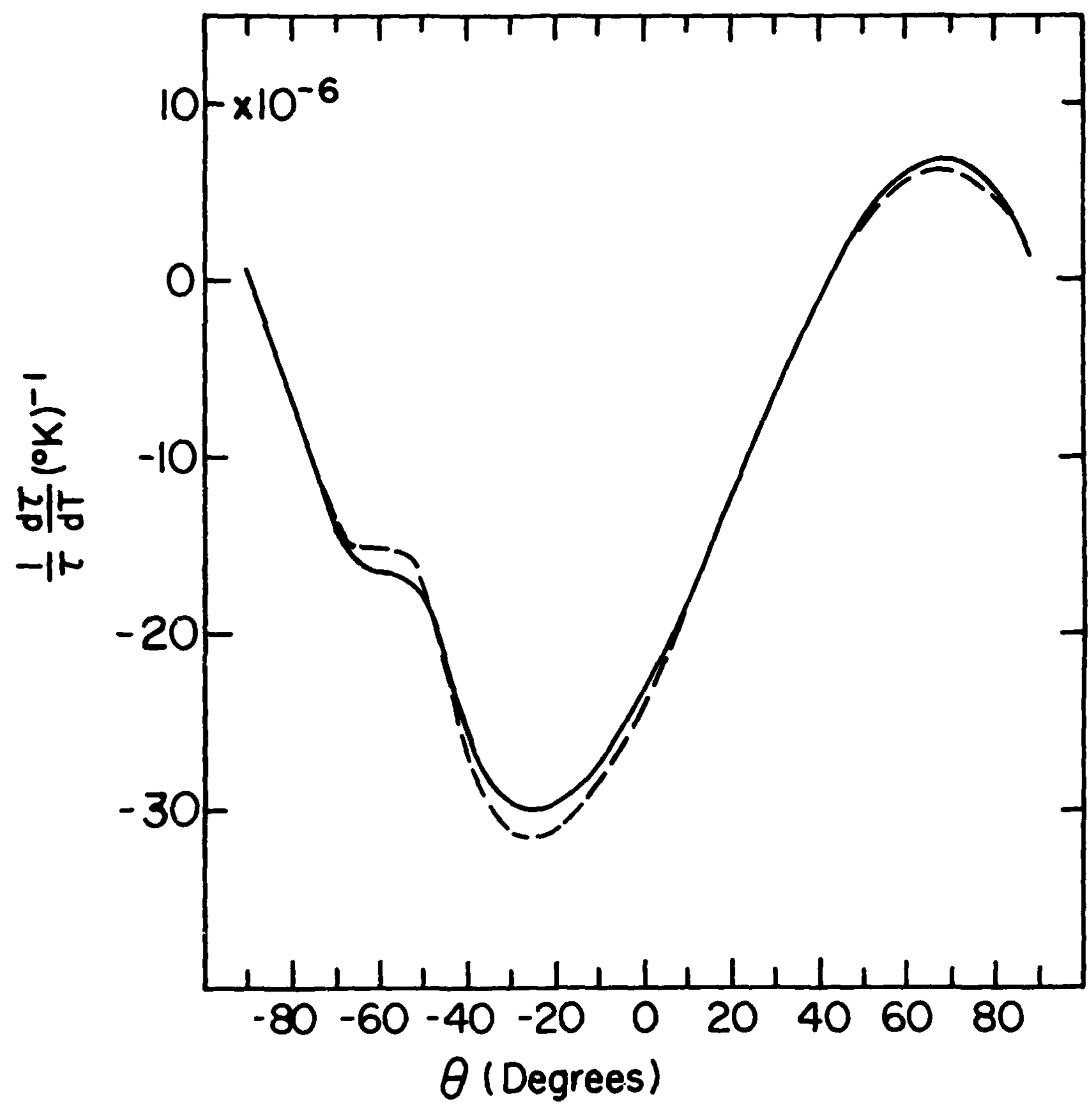
θ (Degrees)

Figure 3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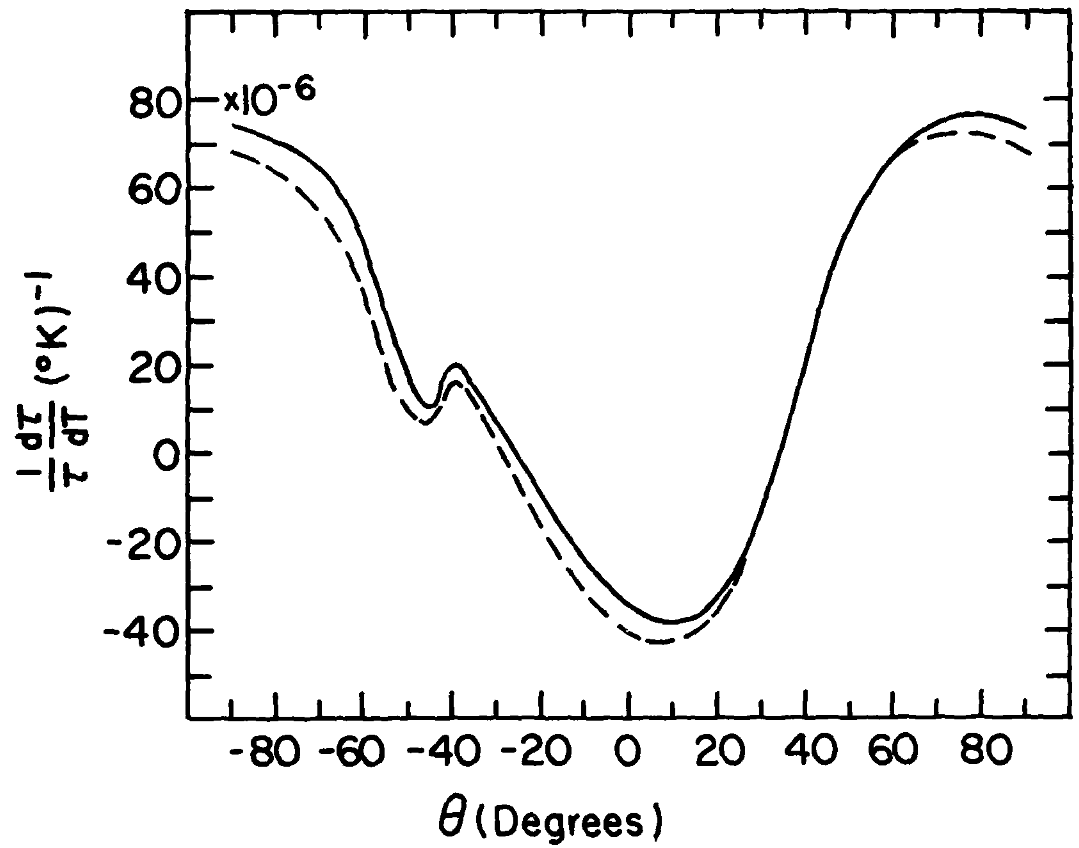
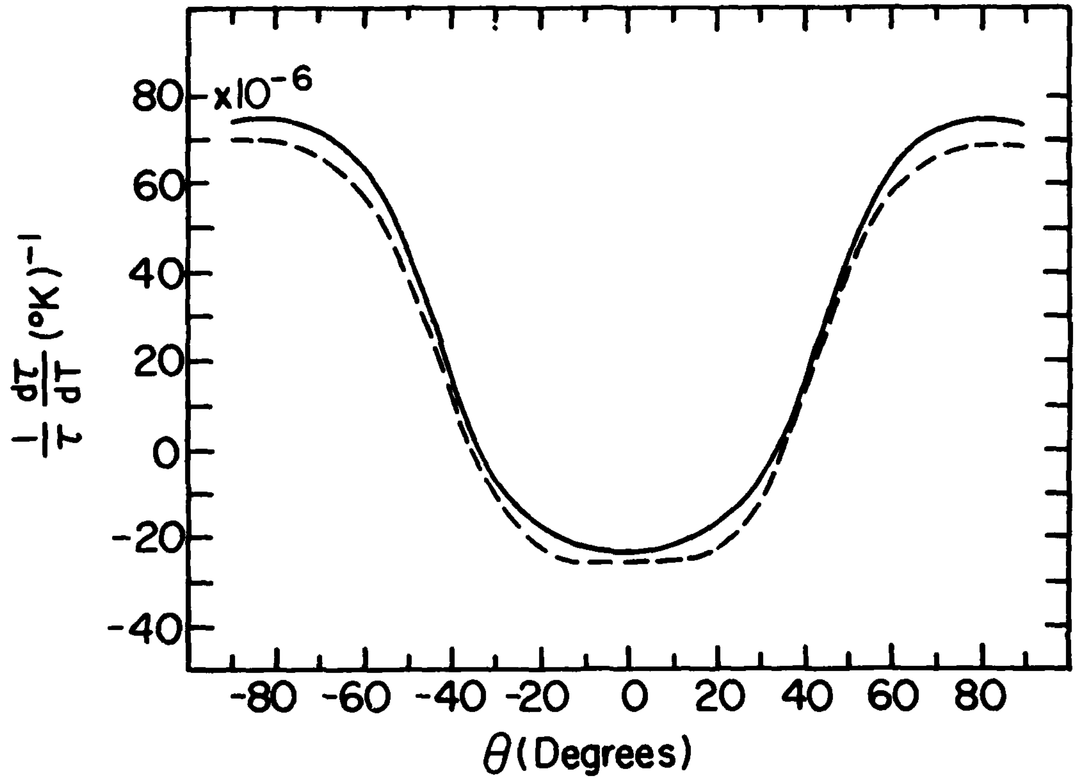
-80 -60 -40 -20 0 20 40 60 80
θ (Degrees)

Figure 4



$\times 10^{-6}$
80
60
40
20
0
-20
-40
$\frac{1}{\tau}\frac{d\tau}{dT}$ $(^{\circ}K)^{-1}$
-80 -60 -40 -20 0 20 40 60 80
θ (Degrees)

Figure 5



80
60
40
20
0
-20
-40
x10^-6
$\frac{1}{\tau}\frac{d\tau}{dT}$ (°K)$^{-1}$
-80 -60 -40 -20 0 20 40 60 80
θ (Degrees)

Figure 6

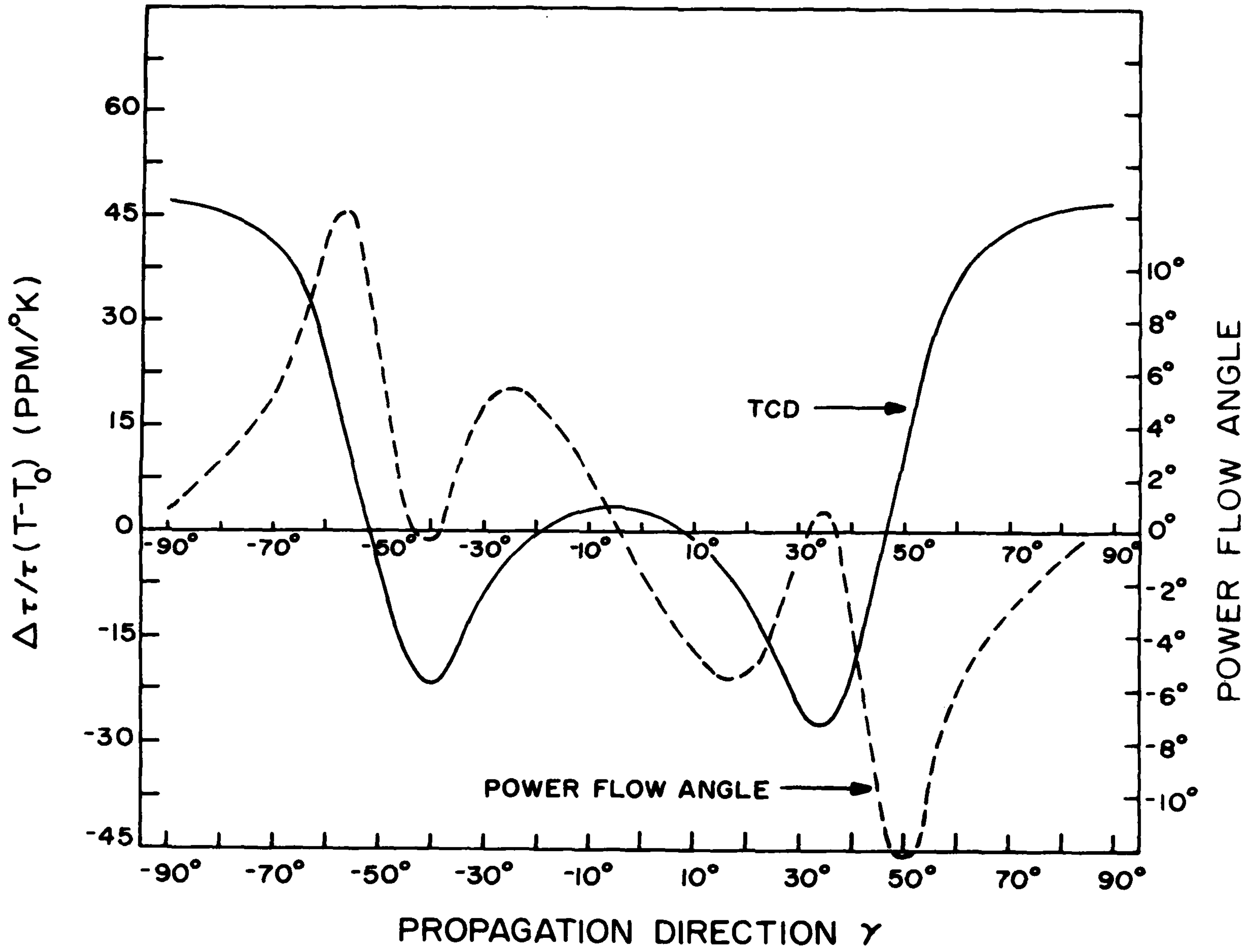
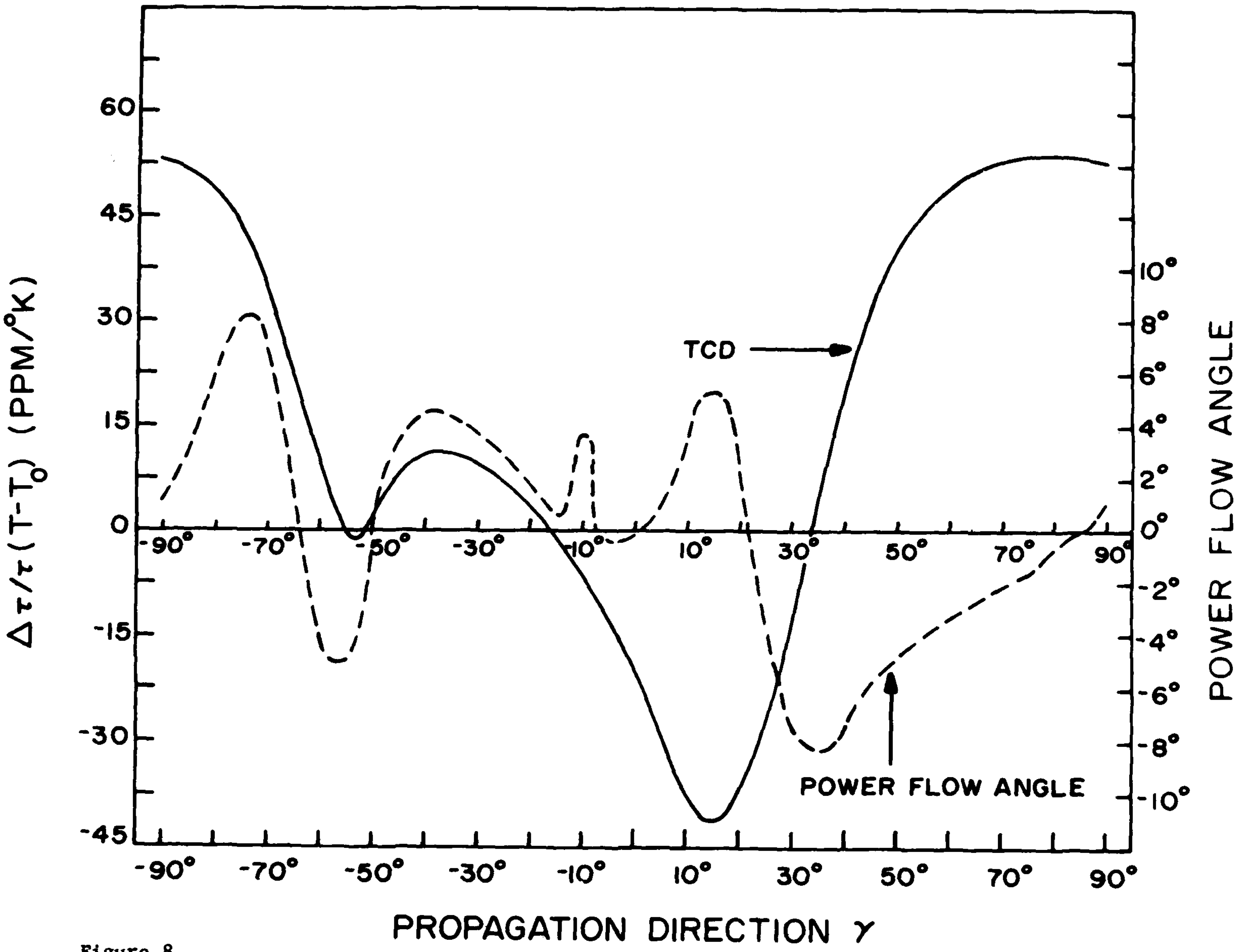

$\Delta\tau/\tau(T-T_0)$ (PPM/°K)
60
45
30
15
0
-15
-30
-45
POWER FLOW ANGLE
10°
8°
6°
4°
2°
0°
-2°
-4°
-6°
-8°
-10°
TCD
POWER FLOW ANGLE
-90°
-70°
-50°
-30°
-10°
10°
30°
50°
70°
90°
PROPAGATION DIRECTION γ

Figure 7



60
45
30
15
0
-15
-30
-45
-90°
-70°
-50°
-30°
-10°
10°
30°
50°
70°
90°
10°
8°
6°
4°
2°
0°
-2°
-4°
-6°
-8°
-10°
TCD
POWER FLOW ANGLE
$\Delta\tau/\tau(T-T_0)$ (PPM/°K)
PROPAGATION DIRECTION γ

Figure 8

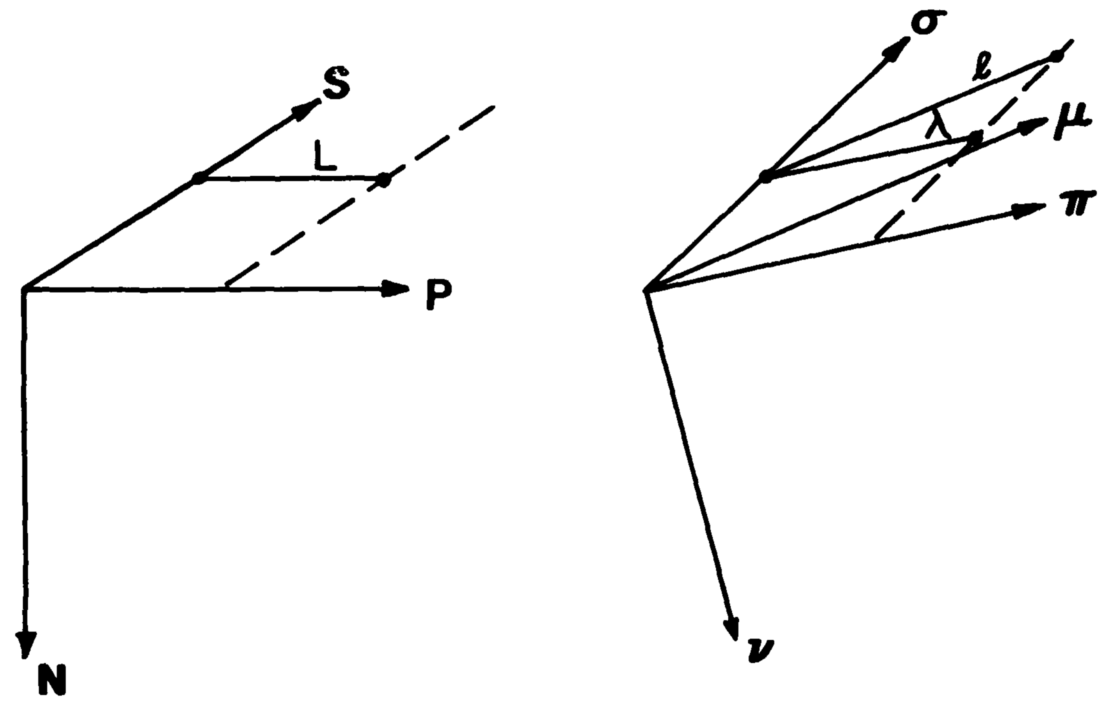


Figure 9

APPENDIX

In this Appendix we derive the relation between the change in actual and change in natural velocity for surface waves. The procedure employed follows that of Thurston[19] who treated plane waves. The treatment of the surface wave case is a bit more cumbersome because of the existence of the free surface. Consequently, in this Appendix the distinction between capital Latin and lower case Greek indices is important.

Let ν_γ and σ_γ be unit vectors normal to the surface and in the direction of the phase fronts of the surface wave, respectively, both at temperature T, as shown in Fig.9. Clearly, then the propagation direction π_γ is given by

$$\underset{\sim}{\pi} = \underset{\sim}{\nu} \times \underset{\sim}{\sigma}, \quad \pi_\gamma = \epsilon_{\gamma\alpha\beta}\nu_\alpha\sigma_\beta, \tag{A1}$$

where $\epsilon_{\gamma\alpha\beta}$ is a skew-symmetric tensor. In the undeformed state at temperature T_o the unit vectors $\underset{\sim}{\nu}$ and $\underset{\sim}{\sigma}$ were in the directions $\underset{\sim}{N}$ and $\underset{\sim}{S}$, respectively, in Fig.9. We denote the vector orthogonal to $\underset{\sim}{N}$ and $\underset{\sim}{S}$ by $\underset{\sim}{P}$, which is in the <u>natural</u> propagation direction, and we write

$$\underset{\sim}{P} = \underset{\sim}{N} \times \underset{\sim}{S}, \quad P_K = E_{KLM}N_L S_M, \tag{A2}$$

where E_{KLM} is a skew-symmetric tensor. In the deformation accompanying the temperature change from T_o to T the line element in the direction $\underset{\sim}{P}$ at T_o takes the direction $\underset{\sim}{\mu}$ at T, which in general differs from the direction $\underset{\sim}{\pi}$. The intermediate directions $\underset{\sim}{\nu}$, $\underset{\sim}{\sigma}$ and $\underset{\sim}{\mu}$ may be expressed in terms of the reference directions $\underset{\sim}{N}$, $\underset{\sim}{S}$ and $\underset{\sim}{P}$ by means of the relations[20]

$$\nu_\gamma = \frac{N_L X_{L,\gamma}}{B^{(N)}}, \quad \sigma_\gamma = \frac{\xi_{\gamma,L}S_L}{C^{(S)}}, \quad \mu_\gamma = \frac{\xi_{\gamma,L}P_L}{C^{(P)}}, \tag{A3}$$

where

$$B^{(N)} = \sqrt{N_K C^{-1}_{KM} N_M}\ ,\quad C^{(S)} = \sqrt{S_K C_{KM} S_M}\ ,\quad C^{(P)} = \sqrt{P_K C_{KM} P_M}\ , \tag{A4}$$

and

$$C_{KM} = \xi_{\alpha,K}\xi_{\alpha,M}\ ,\quad C^{-1}_{KM} = X_{K,\alpha}X_{M,\alpha}\ , \tag{A5}$$

where $\xi_\gamma = \xi_\gamma(X_R)$ is the intermediate coordinate at T and is related to the quantities appearing in (2.4) by

$$\xi_\alpha = \delta_{\alpha L}(X_L + w_L)\ , \tag{A6}$$

where $\delta_{\alpha L}$ is a Kronecker delta, which is required for consistency with the notation employed[5], and a comma followed by a Greek index, say β, denotes partial differentiation with respect to the intermediate coordinate ξ_β. Substituting from (A3) into (A1), we obtain

$$\pi_\gamma = e_{\gamma\alpha\beta} N_L X_{L,\alpha} \xi_{\beta,K} S_K / B^{(N)} C^{(S)}\ , \tag{A7}$$

which is clearly different from $(A3)_3$.

Since the natural (or reference) wavelength L is in the direction $\underset{\sim}{P}$, from deformation theory[20] and $(A3)_3$, we have

$$\ell\mu_\gamma = \xi_{\gamma,K} P_K L\ , \tag{A8}$$

where ℓ is the deformed length that was L when undeformed and

$$\ell = C^{(P)} L\ . \tag{A9}$$

Inasmuch as L is the undeformed (or reference) distance between two phase fronts, ℓ is the deformed line between the same two (deformed) phase fronts, which, however, is not normal to the phase fronts because

$$\underset{\sim}{\mu} \cdot \underset{\sim}{\sigma} \neq 0\ . \tag{A10}$$

Since the distance between two deformed phase fronts normal to the phase fronts is the actual wavelength λ, the component of $\ell\underset{\sim}{\mu}$ in the direction $\underset{\sim}{\pi}$ is the actual wavelength[19], and we have

$$\lambda = \ell \underset{\sim}{\mu} \cdot \underset{\sim}{\pi} . \tag{A11}$$

Substituting from (A7) and (A8) into (A11) and rearranging terms, we obtain

$$\lambda = L e_{\gamma\alpha\beta} \xi_{\beta,K} \xi_{\gamma,M} P_M S_K N_L X_{L,\alpha} / B^{(N)} C^{(S)} . \tag{A12}$$

Utilizing the well-known relations[20]

$$e_{\delta\beta\gamma} \xi_{\delta,R} \xi_{\beta,K} \xi_{\gamma,M} = J^1 E_{RKM}, \quad \xi_{\delta,R} X_{R,\alpha} = \delta_{\alpha\delta} , \tag{A13}$$

we obtain

$$e_{\alpha\beta\gamma} \xi_{\beta,K} \xi_{\gamma,M} = J^1 X_{R,\alpha} E_{RKM}, \tag{A14}$$

where, of course

$$J^1 = \det \xi_{\zeta,T} . \tag{A15}$$

Substituting from (A14) into (A12) and rearranging terms, we obtain

$$\lambda = L J^1 E_{RKM} S_K P_M X_{R,\alpha} X_{L,\alpha} N_L / B^{(N)} C^{(S)} , \tag{A16}$$

which, with

$$\underset{\sim}{S} \times \underset{\sim}{P} = \underset{\sim}{N}, \quad E_{RKM} S_K P_M = N_R, \tag{A17}$$

and $(A4)_1$ and $(A5)_2$, yields

$$\lambda = L J^1 B^{(N)} / C^{(S)} , \tag{A18}$$

which holds for arbitrarily large deformations. Before reducing to infinitesimal deformations we record the well-known relation

$$E^1_{KM} = \frac{1}{2} (C_{KM} - \delta_{KM}) . \tag{A19}$$

When the deformation is infinitesimal and terms higher than linear in $w_{L,R}$ may be ignored, from (A5), (A6) and (A19) we obtain

$$E^{1}_{KM} = \frac{1}{2}\,(w_{K,M} + w_{M,K})\,, \tag{A20}$$

$$C_{KM} = \delta_{KM} + 2E^{1}_{KM}\,, \quad C^{-1}_{KM} = \delta_{KM} - 2E^{1}_{KM}\,. \tag{A21}$$

Substituting from (A21) into (A4), utilizing the fact that $\underset{\sim}{N}$, $\underset{\sim}{S}$ and $\underset{\sim}{P}$ are unit vectors and making a Taylor expansion of the radicals while retaining only linear terms, we obtain

$$B^{(N)} = 1 - E^{(N)1}\,, \quad C^{(S)} = 1 + E^{(S)1}\,, \quad C^{(P)} = 1 + E^{(P)1}\,, \tag{A22}$$

where

$$E^{(N)1} = N_K E^{1}_{KM} N_M\,, \quad E^{(S)1} = S_K E^{1}_{KM} S_M\,, \quad E^{(P)1} = P_K E^{1}_{KM} P_M\,, \tag{A23}$$

and $E^{(N)1}$, $E^{(S)1}$ and $E^{(P)1}$ are the extensional strains in the directions $\underset{\sim}{N}$, $\underset{\sim}{S}$ and $\underset{\sim}{P}$, respectively. Similarly, substituting from (A6) into (A15), expanding, retaining only linear terms in $w_{L,R}$ and employing (A20), the invariance of the trace of the strain tensor and (A23), we obtain

$$J^{1} = 1 + E^{(N)1} + E^{(S)1} + E^{(P)1}\,. \tag{A24}$$

Finally, substituting from (A22) and (A24) into (A18), making a Taylor expansion in the $E^{(S)1}$, multiplying out and retaining only linear terms in the $E^{(\cdot)1}$, we obtain

$$\lambda = L(1 + E^{(P)1})\,, \tag{A25}$$

which is the equation that has always been employed without a complete demonstration of its validity. Since the frequency is the same whether the reference coordinates $\underset{\sim}{X}$ at T_o or the deformed coordinates $\underset{\sim}{\xi}$ at T are employed, we have

$$V_a/\lambda = V/L\,, \tag{A26}$$

where V_a is the actual velocity and V is the natural velocity of the surface wave. Since

$$V_a = V_1 + \Delta V_a , \quad V = V_1 + \Delta V , \tag{A27}$$

where V_1 is the unperturbed velocity at T_o and both ΔV_a and ΔV are small compared with V_1, from (A25) - (A27), we have

$$\frac{\Delta V_a}{V_1} = \frac{\Delta V}{V_1} + E^{(P)1} , \tag{A28}$$

because $E^{(P)1} \ll 1$. Equation (A28) is Eq.(4.4).

Since we have presented the treatment in this Appendix leading to Eq.(A18), from which we have obtained the reduced form (A25) for the case of infinitesimal strain, for completeness we record without proof the limiting forms of $\underset{\sim}{\nu}$, $\underset{\sim}{\sigma}$, $\underset{\sim}{\mu}$ and $\underset{\sim}{\pi}$ for the case of infinitesimal strain. When we ignore the distinction between lower case Greek and capital Latin indices these forms are;

$$\nu_L = N_K[\delta_{KL}(1 + E^{(N)1}) - w_{K,L}] ,$$

$$\sigma_L = S_K[\delta_{KL}(1 - E^{(S)1}) + w_{L,K}] ,$$

$$\mu_L = P_K[\delta_{KL}(1 - E^{(P)1}) + w_{L,K}] ,$$

$$\pi_L = P_L - S_L S_M w_{K,M} P_K + N_L N_M w_{M,K} P_K . \tag{A29}$$